EUF - 2013 - 1sem 22/03/2016

Q1. a) $r > R$

$$E = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0} = \boxed{\frac{Q}{4\pi r^2 \varepsilon_0}}$$

$Q_{enc} = Q$

$r \leq R$

$$E = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0} = \boxed{\frac{rQ}{4\pi R^3 \varepsilon_0}}$$

$$Q_{enc} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} Q$$

b) $dF = q \cdot \frac{dV \cdot Q}{4\pi r^2 \varepsilon_0 V}$?

c) $F = \frac{q \cdot Q}{2} \int_0^R \frac{4\pi r^2 \, dr}{4\pi r^2 \varepsilon_0 \cdot 4\pi R^3} = \frac{qQ}{2\varepsilon_0 R^2}$?

Q2.

a) $\sigma = \frac{q}{A}$

$$E = \frac{\sigma}{\varepsilon_0} \hat{n} = \frac{q_0 \sin(\omega t)}{\pi a^2 \varepsilon_0} \hat{z}$$

b) $\bar{\nabla} \times E = -\frac{\partial E_z}{\partial r} \hat{\phi} = 0 \Rightarrow \int H \, dl = \frac{\partial}{\partial t} \int D \, ds \rightarrow B = \frac{\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \cdot \pi r^2}{2\pi r}$

$$\boxed{B = \frac{\mu_0 q_0 \cos(\omega t) \cdot \omega r}{2\pi a^2} \hat{\phi}}$$

c) $S = \frac{(E \times B)}{\mu_0} \Rightarrow \frac{E_z \cdot B_\phi}{\mu_0} (-\hat{r}) = \left(\frac{q_0 \sin(\omega t)}{\pi a^2 \varepsilon_0}\right) \cdot \left(\frac{q_0 \cos(\omega t) \cdot \omega r}{2\pi a^2}\right) (-\hat{r})$ ✓

$$E \times B = \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ E_r & E_\phi & E_z \\ B_r & B_\phi & B_z \end{vmatrix} = -E_z B_\phi \hat{r}$$

$$\boxed{S = \frac{q_0^2 \omega r \sin(\omega t)\cos(\omega t)}{2\pi^2 a^4 \varepsilon_0} (-\hat{r})} = \frac{q_0^2 \omega r \sin(2\omega t)}{(2\pi a)^2 \varepsilon_0} ✓$$

d) $\frac{\omega a}{c} \ll 1 \qquad u = \frac{1}{2}\left(\frac{\varepsilon_0 q_0^2 \sin^2(\omega t)}{(\pi a^2 \varepsilon_0)^2} + \frac{\mu_0 q_0^2 \omega^2 r^2 \cos^2(\omega t)}{(2\pi a^2)^2 \mu_0}\right) = \frac{q_0^2 \mu_0}{2\pi^2 a^4}\left[c^2 \sin^2(\omega t) + \frac{\omega^2 r^2 \cos^2(\omega t)}{4}\right]$

$$\frac{\partial u}{\partial t} = \frac{q_0^2 \mu_0}{2\pi^2 a^4}\left[c^2 \cdot 2 \underbrace{\sin(\omega t) \cdot \cos(\omega t)}_{\frac{1}{2}\sin(2\omega t)} \cdot \omega + \underbrace{\frac{\omega^2 r^2}{4} \cdot 2\cos(\omega t) \cdot (-\sin(\omega t) \cdot \omega)}_{2}\right]$$

$$\nabla S = \frac{1}{r}\frac{\partial (r S_r)}{\partial r} = \frac{1}{r} \cdot \frac{q_0^2 \omega r \sin(\omega t)\cos(\omega t)}{\pi^2 a^4 \varepsilon_0} = \frac{q_0^2 \omega \sin(2\omega t)}{2\pi^2 a^4 \varepsilon_0}$$

$$\nabla S + \frac{\partial u}{\partial t} = 0 \Rightarrow \frac{q_0^2 \omega \sin(2\omega t)}{2\pi^2 a^4 \varepsilon_0} + \frac{q_0^2 \mu_0 \sin(2\omega t)\omega}{2\pi^2 a^4}\left[c^2 - \frac{\omega^2 r^2}{4}\right]$$

EUF 2013 - 1Sem 10.22

pt2

$$\frac{q_0^2\,\omega \sin(2\omega t)}{2\pi^2 c^4 \varepsilon_0} + \frac{q_0\,\omega \sin(2\omega t)}{2\pi a^4 \varepsilon_0}\left[1-\left(\frac{\omega r}{4c}\right)^2\right] = 0 \quad ✓$$

Q3.

a) $$|\langle\psi|\psi\rangle|^2 = 1 \Rightarrow \int_{-\infty}^{\infty} |\psi(x)|^2\,dx = 1 \Rightarrow \int_{-\frac{L}{2}}^{\frac{L}{2}} A^2\cos^2\left(\frac{3\pi x}{L}\right)dx = 1$$

$$A^2\left[\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{2} + \frac{\cos}{2}\left(\frac{6\pi x}{L}\right)dx\right] = 1 \Rightarrow A^2\cdot\frac{L}{2} = 1 \Rightarrow \boxed{A = \sqrt{\frac{2}{L}}}$$

b) $$\int_{-\frac{L}{4}}^{\frac{L}{4}} \frac{2}{L}\cdot\left(\frac{1}{2} + \frac{\cos}{2}\left(\frac{6\pi x}{L}\right)\right)dx = \frac{1}{2} + \frac{2}{L}\left(\frac{L}{12\pi}\sin\left(\frac{6\pi x}{L}\right)\right)\Bigg|_{-\frac{L}{4}}^{\frac{L}{4}}$$

$$= \frac{1}{2} + \frac{1}{6\pi}\left[\overset{-1}{\sin\left(\frac{3\pi}{2}\right)} - \overset{-1}{\sin\left(-\frac{3\pi}{2}\right)}\right] = \frac{1}{2} - \frac{1}{3\pi}$$

c) $$\frac{\partial\psi}{\partial x} = \frac{A\,3\pi}{L}\left(-\sin\left(\frac{3\pi x}{L}\right)\right)$$

$$\frac{\partial^2\psi}{\partial x^2} = -A\left(\frac{3\pi}{L}\right)^2\cos\left(\frac{3\pi x}{L}\right) = -\left(\frac{3\pi}{L}\right)^2\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{\partial \Psi}{\partial x^2} = E\,\Psi(x) \Rightarrow \boxed{+\frac{\hbar^2}{2m}\left(\frac{3\pi}{L^2}\right)^2 = E}$$

d) $$k^2 = \frac{2mE}{\hbar^2} \Rightarrow k = \frac{n\pi}{L} \Rightarrow E = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

$$E_0 = \frac{\hbar^2\pi^2}{2mL^2} \Rightarrow \Delta E = E_3 - E_0 = \left(\frac{\hbar\pi}{L}\right)^2 \frac{1}{2m}\left[9-1\right] = \frac{4\hbar^2\pi^2}{mL}$$

$$\hbar = \frac{h}{2\pi}$$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = hc \cdot \frac{mL}{4\pi^2\hbar^2} \Rightarrow \boxed{\lambda = \frac{mLc}{2\pi\hbar}}$$

10,5

Q4. $$\frac{dN(t)}{dt} = -RN(t)$$

a) $$\frac{dN(t)}{N(t)} = -R\,dt \Rightarrow \boxed{N(t) = N(0)\cdot e^{-Rt}}$$

$$\frac{N(\bar{t})}{N(0)} = \frac{1}{e} = e^{-2\cdot 10^{-6}R} \Rightarrow 0 = -2\cdot 10^{-6}R + 1$$

$$\boxed{R = \frac{1}{2}\cdot 10^{6}}\ \frac{1}{s}$$

s

no referencial do muon

e) No referencial da Terra:

$$\bar{t}' = \gamma\,\bar{t} = 35\cdot 10^{-6}\,s$$

$$d = \bar{t}' \cdot v_m = 35\cdot 10^{-6}\cdot 0{,}998\,c \approx 9000\,m$$

b) $$d = \bar{t}\cdot v_m = 2\cdot 10^{-6}\cdot 0{,}998\,c = 600\,m$$

$$N(\bar{t}) = \frac{10^8}{e}$$

EVF 2013. 1sn 2263

12:03

Q5.

a) $E = \frac{p^2}{2m} + mgh$

$$Z = \int d^3p\, d^3x\, e^{-\beta\left(\frac{p^2}{2m} + mgh\right)} = \int_0^L d^3x\, e^{-\beta mgh} \cdot \underbrace{\int_{-\infty}^{\infty} d^3p\, e^{-\beta\frac{p^2}{2m}}}_{\left(\frac{\pi}{\alpha}\right)^{\frac{3}{2}} \Rightarrow \left(\frac{2m\pi}{\beta}\right)^{\frac{3}{2}}}$$

$\alpha = \frac{\beta}{2m}$

$$Z = \left(\frac{2m\pi}{\beta}\right)^{\frac{3}{2}} \cdot \left(\frac{e^{-\beta mgL} - 1}{-\beta mg}\right)$$

+ details

a)

$$Z = \iiint_0^L \iiint_{-\infty}^{\infty} e^{-\beta\left(\frac{p_x^2 + p_y^2 + p_z^2}{2m}\right) - \beta mgz}\, dp_x dp_y\, dp_z\, dx\, dy\, dz$$

$$Z = \underbrace{\iint_0^L}_{L^2} \underbrace{\iiint_{-\infty}^{\infty} e^{-\beta\left(\frac{p_x^2 + p_y^2 + p_z^2}{2m}\right)} dp_x dp_y dp_z}_{\left(\frac{2m\pi}{\beta}\right)^{\frac{3}{2}}} \underbrace{\int_0^L e^{-\beta mgz}\, dz}$$

$$Z = \left(\frac{2m\pi L^2}{\beta}\right)^{\frac{3}{2}} \cdot \left[\frac{1 - e^{-\beta mgL}}{\beta mgL}\right]$$ ✓

b) $\langle K \rangle = -\frac{\partial \ln Z_K}{\partial \beta} = \frac{3}{2}\frac{\partial}{\partial \beta} \ln \beta = \frac{3}{2}kT$

$\ln Z_K = \frac{3}{2}\left[\ln(2m\pi L^2) - \ln \beta\right]$

5

c) $\langle T \rangle = -\frac{\partial \ln z_p}{\partial \beta} = \frac{-e^{-\beta mgL} \cdot mgL}{1-e^{-\beta mgL}} + \frac{1}{\beta}$

$\ln z_p = +\ln\left(1-e^{-\beta mgL}\right) - \ln\left(\beta mgL\right)$

$= \frac{mgL}{1-e^{\beta mgL}} + \frac{1}{\beta}$

d) $e^{\beta mgL} \approx 1 + \beta mgL + \ldots$

$= \frac{mgL}{\cancel{1}-\cancel{1}-\beta mgL} + \frac{1}{\beta} = 0$

14:33

14:57

$I = \int r^2 dm = \int_{-\frac{D}{2}}^{\frac{D}{2}} r^2 \cdot \rho \cdot dr = \rho \cdot \frac{r^3}{3}\Big|_{-\frac{D}{2}}^{\frac{D}{2}} = \frac{\rho D^3}{12} = \frac{D^2 m}{12}$

Q6.

a) $\tau = I \cdot \alpha = \frac{mD^2}{4} \cdot$ (acc ang) $\quad \omega^2 = \frac{v^2}{r^2} \quad dm = dr\rho$

$\alpha = \frac{d\omega}{dt}$ $\quad F = ma = \frac{mv^2}{r}$ $\quad$ densidade linear

$\tau = dF = d \cdot \frac{m\omega^2 D r}{2}$

Q8. $|\Psi(0)\rangle = \sum_n c_n |\phi_n\rangle$

a) $\boxed{\sum_n |c_n|^2 = 1}$

$|\Psi(t)\rangle = \sum_n c_n |\phi_n\rangle \cdot e^{-i\frac{E_n}{\hbar}t}$

Prob $n \geq 1$ =

$\boxed{= \sum_{n=2}^{\infty} |c_n|^2 - |c_0|^2 - |c_1|^2}$

b) $|\Psi(0)\rangle = c_0|\phi_0\rangle + c_1|\phi_1\rangle$

$\langle H\rangle = \langle\Psi(0)|\hat{H}|\Psi(0)\rangle = (\langle\phi_0|c_0 + \langle\phi_1|c_1)\, \underbrace{\hat{H}\,(c_0|\phi_0\rangle + c_1|\phi_1\rangle)}_{E_0c_0|\phi_0\rangle + E_1c_1|\phi_1\rangle}$

$\boxed{= E_0 c_0^2 + E_1 c_1^2}$

$E_0 = \frac{\hbar\omega}{2}, \quad E_1 = \frac{3\hbar\omega}{2}$

$\langle H\rangle = \hbar\omega \Rightarrow E_0|c_0|^2 + E_1|c_1|^2 = \hbar\omega \Rightarrow \frac{|c_0|^2}{2} + \frac{3|c_1|^2}{2} = 1$

$|c_0|^2 + |c_1|^2 = 1$

$|c_0|^2 = 1 - |c_1|^2$

$\frac{3|c_1|^2}{2} = 1 - \frac{(1-|c_1|^2)}{2}$

$3|c_1|^2 = 2 - 1 - |c_1|^2 \Rightarrow \boxed{|c_1|^2 = \frac{1}{4}}$

$\boxed{|c_0|^2 = \frac{3}{4}}$

$v = \omega r$

Q7. a) $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m[\dot{r}^2 + r^2\omega^2] = L$

$x = r\cos(\omega t) \quad \dot{x} = \dot{r}\cos(\omega t) - r\sin(\omega t)\cdot\omega$

$y = r\sin(\omega t)$

b) $L \Rightarrow m\ddot{r} - mr\omega^2 = 0$

$\boxed{\ddot{r} = r\omega^2}$

$p_k = \frac{\partial L}{\partial \dot{q}_k}$

c) $r = e^{\pm\omega t}$ $(r_0?)$

$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}$

$\dot{r} = \frac{p_r}{m}$

d) $H = \sum p_k \dot{q}_k - L$

$H = m\dot{r}^2 - \frac{1}{2}m\dot{r}^2 - r^2\omega^2 = \frac{1}{2}m\dot{r}^2 - r^2\omega^2$

$H = \frac{1}{2}\frac{p_r^2}{2m} - r^2\omega^2; \quad \boxed{\dot{r} = \frac{\partial H}{\partial p_r} = \frac{p_r}{m} \quad \dot{p}_r = -\frac{\partial H}{\partial r} = 2r\omega}$

e) $\frac{\partial L}{\partial t} = 0$ energy conserved

$\frac{\partial H}{\partial t} = 0$ N cons.

c) $\langle\Psi(0)|\hat{X}|\Psi(0)\rangle = (\langle\phi_0|c_0 + \langle\phi_1|c_1^*)\,\hat{X}\,(c_0|\phi_0\rangle + c_1|\phi_1\rangle) = \langle\hat{X}\rangle$

c_0 = real & posit.

$c_1 = |c_1|e^{i\theta_1}$

$$(\langle\phi_0|c_0 + \langle\phi_1|c_1^*)\cdot\left[\left(c_0\sqrt{\frac{\hbar}{2m\omega}}\sqrt{0+1}\,|\phi_1\rangle\right) + c_1\sqrt{\frac{\hbar}{2m\omega}}\sqrt{1}\,|\phi_0\rangle\right]$$

$$c_0c_1\sqrt{\frac{\hbar}{2m\omega}} + c_0c_1^*\sqrt{\frac{\hbar(0+1)}{2m\omega}} = \frac{1}{2}\sqrt{\frac{\hbar}{m\omega}}$$

$$c_0\cdot|c_1|e^{i\theta_1}\sqrt{\frac{1}{2}} + c_0|c_1|e^{-i\theta_1}\sqrt{\frac{0+1}{2}} = \frac{1}{2}$$

$|c_0| = \frac{\sqrt{3}}{2}$ $|c_1| = \frac{1}{2}$

$c_0 \cdot c_1 = \frac{\sqrt{3}}{4}$

$$\sqrt{\frac{1}{2}}e^{i\theta_1} + e^{-i\theta_1}\sqrt{\frac{0+1}{2}} = \frac{1}{c_0|c_1|2} \Rightarrow e^{i\theta_1} + e^{-i\theta_1} = \frac{\sqrt{2}}{2c_0|c_1|}$$

$$\cos(\theta_1) = \sqrt{\frac{2}{3}} \Leftrightarrow \theta_1 = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right) \quad 35^\circ$$

d) $|\Psi(0)\rangle = \frac{\sqrt{3}}{2}|\phi_0\rangle + \frac{1}{2}\cdot e^{i\theta_1(0)}|\phi_1\rangle$

$$|\Psi(t)\rangle = \frac{\sqrt{3}}{2}\cdot e^{i\frac{\omega t}{2}}|\phi_0\rangle + \frac{1}{2}e^{i(3\omega\frac{t}{2}+\theta_1(t))}|\phi_1\rangle$$

$\langle\hat{X}\rangle(t) \Rightarrow \langle\Psi(t)|\hat{X}(t)|\Psi(t)\rangle =$

EUF 2013 - 1sem 22/03/2016

Q9.

$\varepsilon_{ijk} = 1$ or -1 if cycled index, or 0 if repeated index

a) $[L_i, L_j] = i\hbar\, \varepsilon_k\, \varepsilon_{ijk} L_k$

$$\vec{L}\times\vec{L} = \begin{vmatrix} i & j & k \\ L_i & L_j & L_k \\ L_i & L_j & L_k \end{vmatrix} = (L_j L_k - L_k L_j)\hat{i} + (L_k L_i - L_i L_k)\hat{j} + (L_i L_j - L_j L_i)\hat{k}$$

$$[L_j, L_k]\hat{i} + [L_k, L_i]\hat{j} + [L_i, L_j]\hat{k}$$

$$= i\hbar\Big[\underbrace{\sum_i^{1} \varepsilon_{ijk} L_i + \sum_j^{1} \varepsilon_{ijk} L_j + \sum_k^{1} \varepsilon_{ijk} L_k}_{\varepsilon_{jjk},\ \varepsilon_{kjk}\ 0}\Big] = i\hbar[L_i + L_j + L_k] = \underline{i\hbar\vec{L}}$$

b) $\vec{L} = R\times P$, $[R_i, P_j] = i\hbar\,\delta_{ij}$

$$[L_i, R_j] = L_i R_j - R_j L_i = (R_i \times P_i) R_j - R_j (R_i \times P_i)$$

$$L = R\times P = [R_j, P_k]\hat{i} + [R_k, P_i]\hat{j} + [R_i, P_j]\hat{k}$$

$$L = i\hbar[\delta_{jk}\hat{i} + \delta_{ki}\hat{j} + \delta_{ij}\hat{k}]$$

Q6.

a) $P = \frac{aT^4}{3}$, $U = aT^4 V$, $T_0 V_0$ $dU = dQ - dW$

$\Delta U = aT_0^4 V_0$ $\Delta U = Q - W$

$W = p\Delta V$

$\boxed{W = \frac{aT_0^4}{3} \cdot V_0}$ vin [illegible]

$\left(\frac{\partial U}{\partial T}\right)_V = 4aT^3 V$

$\left(\frac{\partial U}{\partial V}\right)_T = aT^4$

b) $Q = aT_0^4 V_0 + \frac{aT_0^4}{3} V_0 = \boxed{\frac{4}{3} T_0^4 V_0}$

c) $0 = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV$

$0 = 4aT^3 V\, dT + \left[aT^4 + \frac{aT^4}{3}\right] dV \Rightarrow -4aT^3 V\, dT = \frac{4}{3} aT^4\, dV$

$\frac{-3dT}{T} = \frac{dV}{V} \Rightarrow -3\ln T = \ln V$ $\boxed{T^{-3} = V}$

$V = \frac{1}{T^3} \Rightarrow VT^3 = 1$

d) $W = -\Delta U = -a\Delta T^4 \Delta V = a(T_f^4 - T_0^4) \cdot V_0$

$W = a\left(\frac{T_0^{12}}{16} - T_0^4\right) V_0 = \frac{aV_0 T_0^{12}}{16} - aT_0$

10.